Reliability and validity of numerical and non-numerical order processing

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Introduction

Background

- Numerical abilities are essential for academic achievement and life success in modern societies[1].
- An often overlooked but important property of numbers is ordinality: the relative position or rank (e.g., 4 comes before 5 but after 3)[2].
- Although a link between ordinality and arithmetic ability is suggested[3,4,5], little is known about the reliability of numerical order processing and its precise relationship to mathematical achievement.

Research questions and hypotheses

- Numerical (symbolic and non-symbolic) and non-numerical (letters of the alphabet) order processing and their associations with arithmetic fluency were examined at two time points (T1, T2).
- A significant correlation between numerical order processing and individual arithmetic test scores was expected for number symbols (Hindu-Arabic numerals).
- Number symbols were expected to explain unique variance in arithmetic fluency over and above non-symbolic numerical (dot-arrays) and symbolic non-numerical order (letters of the alphabet) processing.

Stimuli and Procedure

Subjects

- 32 healthy right-handed adults (18 males, 14 females; mean = 23.53 years; range = 20 to 33 years)

Ordinality Task

- Hindu-Arabic numerals, dot-arrays and letters of the alphabet were presented on a computer screen (see Fig. 1).
- Subjects decided whether the presented triads were in-order (ascending/descending) or in a mixed-order.
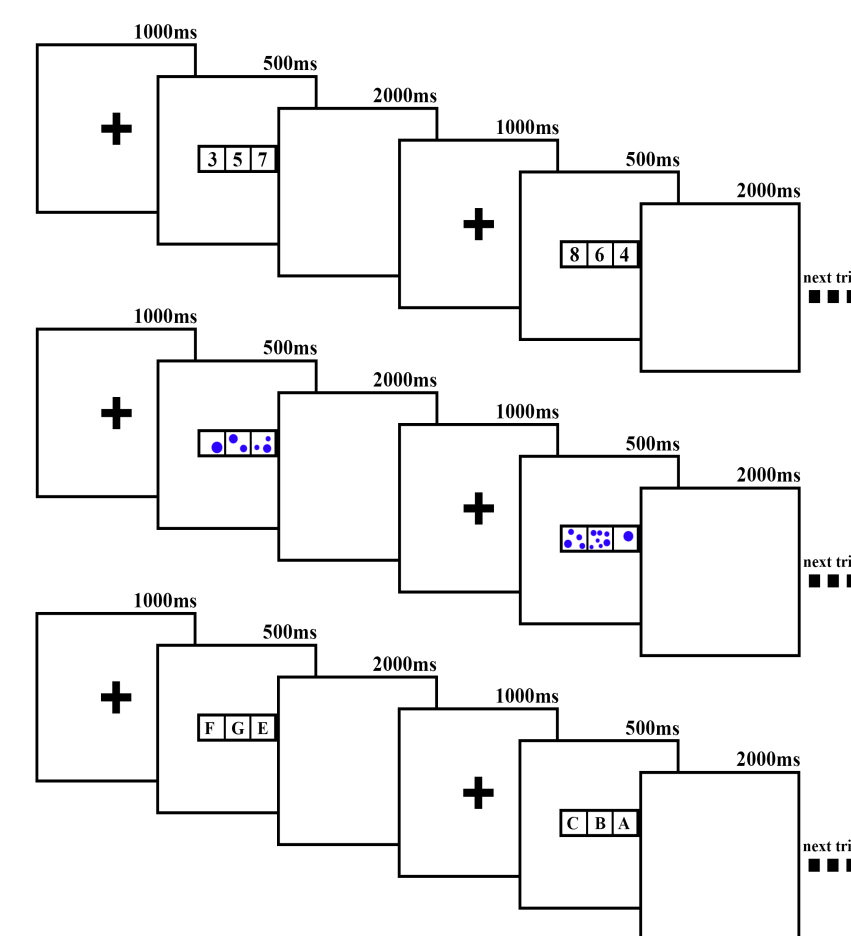

Figure 1: Stimuli

Arithmetic Fluency

- Paper and pencil test of arithmetic fluency (AF) including multiplication, subtraction and addition problems (see Fig. 2).
- Subjects had 90s/120s (easy/difficult condition) per page to solve the tasks.


| 11 - 2 = |
| 8 - 4 = |
| 8 + 3 = |
| 7 + 9 = |
| 5 x 8 = |
| 4 x 2 = |

Figure 2: Calculation problems

Analyses and Results

Reaction Times Ordinality Tasks

Effects	F-value	df	p-value	η_p^2
Hindu-Arabic numerals				
Time-point	20.424	(1, 31)	0.001**	0.397
Order	51.933	(1, 31)	0.001**	0.626
Distance	33.529	(1.663, 51.539)	0.001**	0.520
Time-point x Order	0.300	(1, 31)	0.588	0.010
Time-point x Distance	4.240	(1.891, 58.611)	0.021*	0.120
Order x Distance	35.797	(1.956, 60.625)	0.001**	0.536
Time-point x Order x Distance	3.404	(1.802, 55.873)	0.045*	0.099
Dot-arrays				
Time-point	23.361	(1, 31)	0.001**	0.430
Order	16.075	(1, 31)	0.001**	0.341
Distance	30.053	(1.763, 54.665)	0.001**	0.492
Time-point x Order	0.000	(1, 31)	0.986	0.000
Time-point x Distance	0.948	(1.971, 61.093)	0.393	0.030
Order x Distance	1.343	(1.789, 55.462)	0.269	0.042
Time-point x Order x Distance	0.060	(1.881, 58.317)	0.933	0.002
Letters				
Time-point	1.822	(1, 31)	0.187	0.056
Order	11.602	(1, 31)	0.002*	0.272
Distance	15.532	(1.957, 60.671)	0.001**	0.334
Time-point x Order	0.003	(1, 31)	0.959	0.000
Time-point x Distance	4.663	(1.791, 55.528)	0.016*	0.131
Order x Distance	44.745	(1.919, 59.494)	0.001**	0.591
Time-point x Order x Distance	0.221	(1.848, 57.296)	0.785	0.007

- For Hindu-Arabic numerals an Analysis of Variance (ANOVA) revealed a significant reverse distance effect (rDE) for in-order trials at T1, but not at T2; for mixed-order trials a canonical distance effect (cDE) was observed.
- For dot-arrays, the ANOVA revealed cDEs for in-order and mixed-order trials at T1 and T2.
- For letters of the alphabet, the ANVOA revealed a rDE for ordered stimuli, and a cDE for mixed-ordered stimuli at T1 and T2.

a) Hindu-Arabic numerals

b) Dot-arrays

c) Letters of the alphabet

** p < 0.01; * p < 0.05
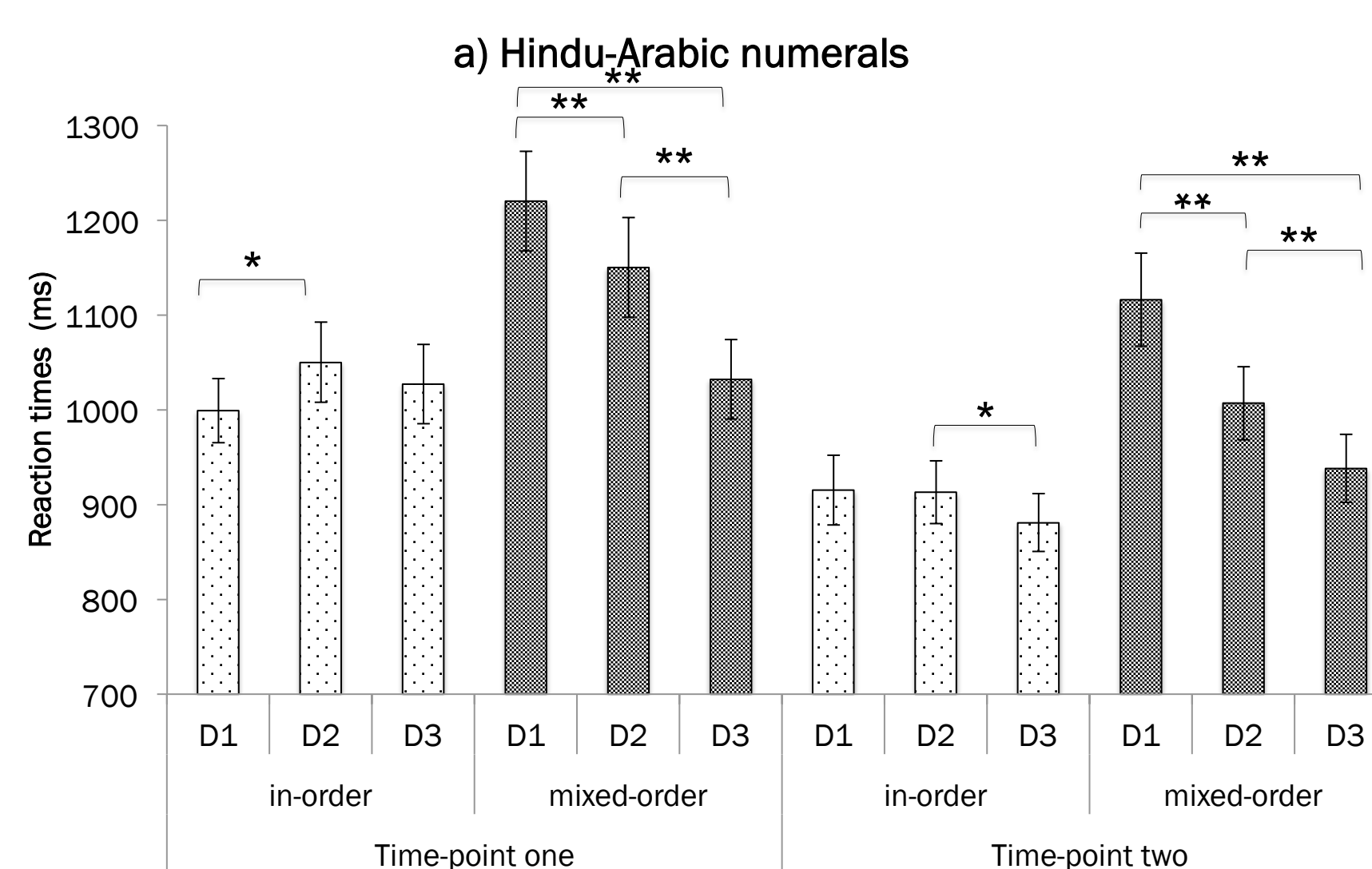
Relationship with Arithmetic Fluency

Arithmetic fluency at T1 and T2 regressed on several in-order measures of numerical and non-numerical ordinality: RT = mean reaction time; DE = distance effect.

Overall model fit: T1: adj. R^2 = 0.315.

Predictor	β	SE	t	p	$r_{partial}$	r
RT Numbers	-0.203	0.068	-3.010**	0.006	-0.516**	-0.563***
DE Numbers	109.173	105.052	1.039	0.309	0.203	-0.217
RT Dots	0.042	0.047	0.879	0.388	0.173	-0.150
DE Dots	-14.515	90.378	-0.161	0.874	-0.032	0.256
RT Letters	-0.038	0.049	-0.780	0.443	-0.154	-0.438*
DE Letters	195.072	100.619	1.939	0.064	0.362	0.096
Constant	405.452	76.446				

Overall model fit: T2: adj. R^2 = 0.202.

Predictor	β	SE	t	p	$r_{partial}$	r
RT Numbers	-0.165	0.079	-2.085	0.047	-0.385*	-0.535**
DE Numbers	-10.209	133.095	-0.077	0.939	-0.015	0.083
RT Dots	0.059	0.063	0.940	0.356	0.185	-0.155
DE Dots	-14.955	85.048	-0.176	0.862	-0.035	0.220
RT Letters	-0.064	0.053	-1.195	0.243	-0.232	-0.476**
DE Letters	42.341	116.190	0.364	0.719	0.073	0.006
Constant	409.824	79.072				

*** p < 0.001; ** p < 0.01; * p < 0.05
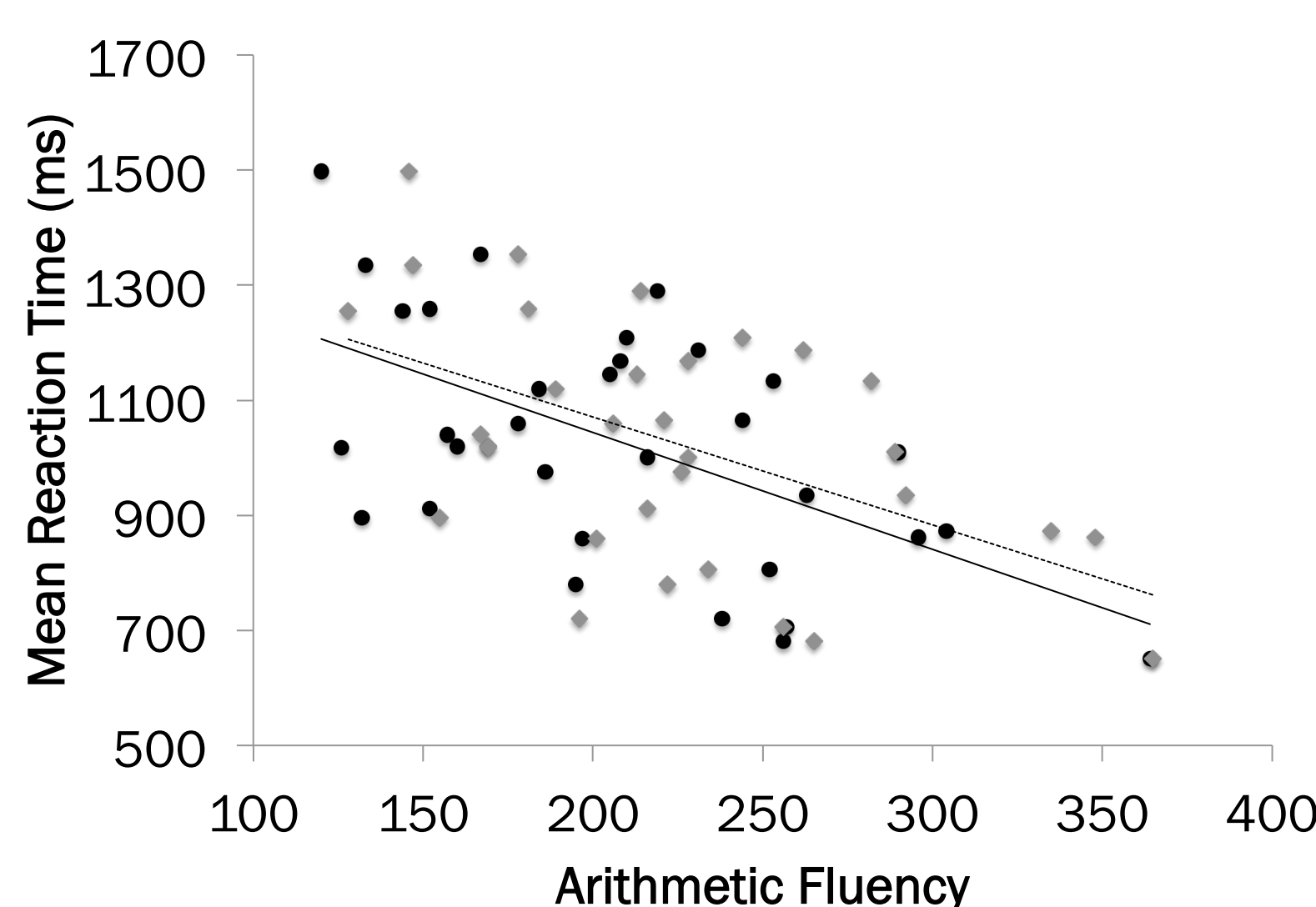
The relationship between ordering of number symbols and arithmetic fluency. Circles represent data collected at T1; Diamonds represent data collected at T2.
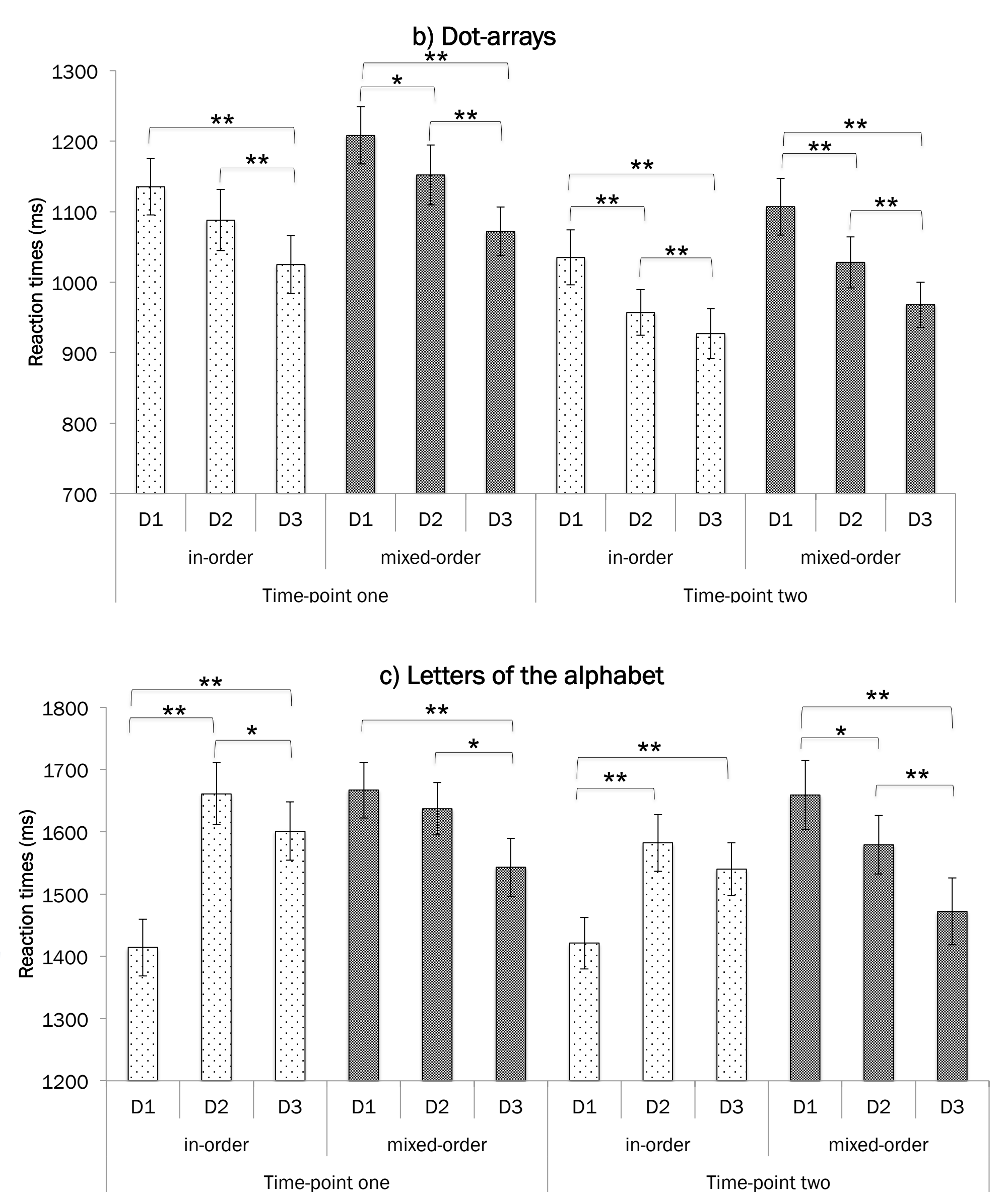
Reliability

Pearson's correlations between T1 and T2 for mean reaction times and distance effects of the ordinal task conditions.

	Numbers	Dot-arrays	Letters
RT in order	.734**	.727**	.732**
DE in order	.373*	.425*	-.025

** p < 0.01; * p < 0.05

Calculation of the distance effects: rDE = (meanRTD2,D3 – meanRTD1) / meanRTD1,D2,D3; and cDE = (meanRTD1 – meanRTD3) / meanRTD1,D3[5].

Discussion

- Distinct behavioural signatures for symbolic (digits and letters) and non-symbolic numerical (dots) ordinality processing support the view that symbolic and non-symbolic ordinal processing engage different cognitive mechanisms.
- Numerical symbolic (digits) ordinality processing has a strong and reliable relationship with arithmetic fluency, and explains a unique portion of variance over and above numerical non-symbolic (dots) and non-numerical (letters) ordinal processing.
- This indicates that the processing of ordinality in numerical symbols is a fundamental property of arithmetic fluency.
- Overall, the present study indicates that the ability to process the ordinal relationship of numerical symbols is a strong and unique predictor of arithmetic fluency, and supports the idea of different cognitive mechanisms underlying this and non-symbolic ordinal processing.

References
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